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On the CSFT approach to localized closed string tachyons

Oren Bergman and Shlomo S. Razamat

Department of Physics, Technion, Israel Institute of Technology
Haifa 32000, Israel
E-mail: bergman@physics.technion.ac.il, razamat@physics.technion.ac.il

Abstract: We compute the potential for localized closed string tachyons in bosonic string theory on the orbifold $\mathbb{C}/\mathbb{Z}_4$ using level-truncated closed string field theory. The critical points of the potential exhibit features which agree with their conjectured identification as lower-order orbifolds. However this case also raises some questions regarding the quantitative predictions associated with these conjectures.

Keywords: Bosonic Strings, Tachyon Condensation, String Field Theory.

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1. Introduction

Though we've learned much about the role of tachyons in open string theory, closed string tachyons have remained somewhat of a mystery. The main difficulty is that closed string tachyons couple directly to gravity and the dilaton, so the back-reaction of the tachyon condensate on the background is important. In an interesting development, Adams, Polchinski and Silverstein (APS) proposed a scenario for the condensation of *localized* closed string tachyons which appear in the twisted sectors of the orbifold $\mathbb{C}/\mathbb{Z}_N$ in type-II superstring theory [1]. This describes a cone with deficit angle $\theta_N = 2\pi(1 - 1/N)$, and the tachyons in question live at the tip of the cone. They conjectured that the condensation of the twisted tachyons affects the background only locally by flattening the cone. The minimum should correspond to flat space, and the other critical points to cones with a smaller deficit angle, namely $\mathbb{C}/\mathbb{Z}_M$ with $M < N$. The conjecture was supported both by a D-brane probe analysis [1, 2] and by worldsheet renormalization group techniques [1, 3, 4, 5]. This background has also been studied from the low-energy supergravity point of view, and was shown to produce radial gravity and dilaton waves which can gradually reduce the deficit angle [6, 7]. A large-N approach to the problem has been discussed in [8, 9]. The relation of localized closed string tachyons to semiclassical instabilities has been addressed in [10].

In a recent paper Okawa and Zwiebach have set out to test the APS conjecture in the context of the bosonic string using closed string field theory [11]. In this case a bulk (untwisted) tachyon exists for all N, but one considers the condensation of just the localized tachyons, which include the twisted tachyons as well as localized modes of the untwisted tachyon. This is similar in spirit to the discussion of open string tachyon condensation in theories containing also closed string tachyons.

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Computation of the tachyon effective potential in open string field theory using level-truncation has lead to several remarkable tests of Sen's conjectures [12, 13] regarding open string tachyon condensation (for a review see [14]). In trying to extend these techniques to closed string tachyons in closed string field theory one encounters two immediate problems. The first is the lack of a clear quantitative prediction. In the open string case Sen's conjecture can be translated into a simple numerical prediction for the tachyon potential, namely that the depth of its minimum is equal to the tension of the original unstable D-brane. Dabholkar has proposed an analogous prediction for the twisted tachyons of $\mathbb{C}/\mathbb{Z}_N$ [15]. From the effective action on the orbifold,

$$S = -\frac{1}{2\kappa_N^2}\int_{\mathbb{R}^{1,7}\times\mathbb{C}/\mathbb{Z}_N} d^{10}x\sqrt{-g}e^{-2\Phi}R - \int_{\mathbb{R}^{1,7}} d^8x\sqrt{-g^{(D-2)}}e^{-2\Phi}V_N(T)\,, \tag{1.1}$$

and assuming that the potential for the twisted tachyons $V_N(T)$ is independent of the metric, one gets a relation between the potential and the deficit angle

$$\theta = \kappa_N^2 V_N(T)\,. \tag{1.2}$$

This means that the relative depths of the various critical points of the potential are given by the difference in deficit angles of the orbifolds they represent. Expressed in terms of the normalized dimensionless potential

$$\widetilde{f}_N(T) \equiv \frac{\kappa_N^2 V_N(T)}{2\pi\left(1-\frac{1}{N}\right)}\,, \tag{1.3}$$

the prediction for the critical point corresponding to $\mathbb{C}/\mathbb{Z}_M$ is therefore

$$\widetilde{f}_N(T_M) = -\frac{\frac{1}{M}-\frac{1}{N}}{1-\frac{1}{N}} \qquad M = 1,\ldots,N-1\,. \tag{1.4}$$

This is also consistent with the fact that the ADM mass of a co-dimension 2 object, like the $\mathbb{C}/\mathbb{Z}_N$ orbifold 7-plane, is given by its deficit angle. However the actual potential depends also on localized modes of the untwisted (bulk) tachyon, and therefore it depends on the metric through the metric dependence of the bulk tachyon expectation values [11]. Strictly speaking therefore, the prediction (1.2) only holds to lowest level, where the massless (level 2) fields do not appear. It does however need to be refined by including, and then integrating out, localized modes of the bulk tachyon. It is not clear how to generalize the prediction to include metric dependence and higher-level fields. The second problem is that, unlike open string field theory, closed string field theory has an infinite number of interaction vertices, so that one would have to include an infinite number of terms in the action at every level [16]. In fact, very little is known about the strength of the interactions beyond the quartic vertex (for which we only have a numerical approximation [17, 18]).

Given these problems, it is quite remarkable how close one gets using a lowest-level truncation and only the cubic vertex [11]. Okawa and Zwiebach computed the localized tachyon potential for $\mathbb{C}/\mathbb{Z}_2$ and $\mathbb{C}/\mathbb{Z}_3$ and found an agreement of about 35% with the

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predicted depth of the minimum of the potential in both cases.[1] For $\mathbb{C}/\mathbb{Z}_3$ there is a second critical point, conjectured to correspond to the decay to $\mathbb{C}/\mathbb{Z}_2$. Its depth was found to be 44% of the predicted value. It was also shown that including the quartic vertex does not change the qualitative features of the potential, although it changes the depths of the critical points somewhat. This suggests that perhaps the higher vertices should be assigned an intrinsic level, and should therefore be truncated according to the level of the approximation.

In this paper we present further tests of this approach to localized tachyon condensation by analyzing the lowest-level localized tachyon potential of $\mathbb{C}/\mathbb{Z}_4$. In this case there are three predictions corresponding to the three possible decay products: flat space, $\mathbb{C}/\mathbb{Z}_2$ and $\mathbb{C}/\mathbb{Z}_3$. The critical points of the potential are arranged on an (irregular) tetrahedron in the three-dimensional space of twisted tachyons (t, t'), where t is complex and t' is real. The maximum, $\mathbb{C}/\mathbb{Z}_4$, is at the body-center, and the four degenerate minima are at the vertices. The minimum is found at 25% of the predicted value for flat space. The $\mathbb{C}/\mathbb{Z}_3$ points are located at the four faces of the tetrahedron, and give 46% of the predicted value. The $\mathbb{C}/\mathbb{Z}_2$ points are located at the six edges, and are actually split into a group of four and a group of two. The former give 39% of the predicted value, and the latter 34%. Technically, the split is due to the fact that the potential does not respect the full tetrahedral symmetry, but only a $\mathbb{Z}_4$ (or really D_4) subgroup, which is the quantum symmetry of the orbifold. It is not clear to us whether the tetrahedral symmetry should be restored at higher level.

In section 2 we review some of the necessary ingredients needed for computing orbifold tachyon potentials. In section 3 we compute the localized tachyon potential for the bosonic string on $\mathbb{C}/\mathbb{Z}_4$, and in section 4 we find the critical points and compare their depths with the predicted values. Section 5 contains our conclusions. We have also included two appendices which contain some details of the computation.

2. Necessary ingredients and general strategy

Let us first review briefly the necessary ingredients for computing localized tachyon potentials in $\mathbb{C}/\mathbb{Z}_N$ in general. This orbifold has $N-1$ twisted sectors corresponding to $N-1$ twist fields σ_k in the orbifold CFT. The conformal dimensions of σ_k are $h_k = \bar{h}_k = \frac{1}{2}\frac{k}{N}\left(1 - \frac{k}{N}\right)$, so the fields σ_k and σ_{N-k} are paired up. The orbifold CFT has a quantum symmetry $\mathbb{Z}_N$ under which the twist fields transform as

$$\sigma_k \to \exp\left(\frac{2\pi k i}{N}\right)\sigma_k \,, \qquad (2.1)$$

and all correlation functions are invariant under this. The untwisted operators $\mathcal{U}_p$ carry momentum p along the directions of the cone and must be projected to the invariant sector:

$$\mathcal{V}_p = \frac{1}{N}\sum_{k=0}^{N-1} \mathcal{U}_{\alpha^k p} \,, \qquad (2.2)$$

[1] An agreement of about 70% was reported in [11] due to an error in identifying the orbifold gravitational coupling κ_N^2 with the flat space gravitational coupling κ^2. This error was also present in an earlier version of this paper.

where $\alpha = \exp(2\pi i/N)$. The basic correlation functions are given by[2] [19]

$$
\begin{aligned}
\langle \sigma_{N-k}(\infty)\sigma_k(0)\rangle &= V_{D-2}\\
\langle \mathcal{V}_p(\infty)\sigma_{N-k}(1)\sigma_k(0)\rangle &= \delta^{-p^2/4}\,V_{D-2}\\
\langle \sigma_{N-2k}(\infty)\sigma_k(1)\sigma_k(0)\rangle &= \sqrt{\frac{\tan(\pi k/N)}{2\pi^2}}\,\frac{\Gamma^2\left(1-\frac{k}{N}\right)}{\Gamma\left(1-\frac{2k}{N}\right)}V_{D-2}\,,
\end{aligned}
\tag{2.3}
$$

where δ is the following function of k/N:

$$
\delta\left(\frac{k}{N}\right) = \exp\left[2\psi(1) - \psi\left(\frac{k}{N}\right) - \psi\left(1-\frac{k}{N}\right)\right], \tag{2.4}
$$

and $\psi(x) = \Gamma'(x)/\Gamma(x)$. Some useful values of this function are $\delta(1/2) = 2^4$, $\delta(1/3) = 3^3$ and $\delta(1/4) = 2^6$. The generalization to include momentum in the seven directions transverse to the cone is straightforward, but does not interest us here since we focus on uniform (in the seven transverse dimensions) tachyon condensation.

The string field theory action to cubic order is given by

$$
S = -2\sum_{\alpha,\beta}(h_\beta - 1)\phi^\alpha m_{\alpha\beta}\phi^\beta - \frac{2\kappa}{3!}\sum_{\alpha,\beta,\gamma}\mathcal{R}^{6-2(h_\alpha+h_\beta+h_\gamma)}\phi^\alpha\phi^\beta\phi^\gamma C_{\alpha\beta\gamma}\,, \tag{2.5}
$$

where κ is the (square root of the) flat space gravitational coupling constant, ϕ^α are the component fields of the string field $|\Psi\rangle = \sum_\alpha c_1\bar{c}_1\phi^\alpha|\mathcal{O}_\alpha\rangle$, and $m_{\alpha\beta}$ and $C_{\alpha\beta\gamma}$ are given by

$$
m_{\alpha\beta} = \langle bpz(\mathcal{O}_\alpha)|c_{-1}\bar{c}_{-1}c_0^- c_0^+ c_1\bar{c}_1|\mathcal{O}_\beta\rangle \tag{2.6}
$$

$$
C_{\alpha\beta\gamma} = \langle c\bar{c}\mathcal{O}_\alpha(0)\, c\bar{c}\mathcal{O}_\beta(1)\, c\bar{c}\mathcal{O}_\gamma(\infty)\rangle\,. \tag{2.7}
$$

The number $\mathcal{R}$ is the inverse of the mapping radius for the map from the punctured unit disk to a punctured 120° wedge of the complex plane, and is given by

$$
\mathcal{R} = \frac{3\sqrt{3}}{4}\,. \tag{2.8}
$$

In particular the action for the bulk tachyon with momentum in the cone directions p, $u(p)c_1\bar{c}_1|p\rangle$, is given by

$$
S_u = -\frac{1}{2}\int\frac{d^2p}{(2\pi)^2}u(-p)(p^2-4)u(p) - \frac{4\kappa}{3!}\int\prod_{i=1}^{3}\left[\frac{d^2p_i}{(2\pi)^2}\mathcal{R}^{2-\frac{1}{2}p_i^2}u(p_i)\right](2\pi)^2\delta^{(2)}(p_1+p_2+p_3)\,. \tag{2.9}
$$

This result is universal for all N. The integrals are over all momenta in $\mathbb{C}$, where one identifies $u(p) = u(\alpha^k p)$. The rest of the tachyon action involves the twisted tachyons and is computed using the correlation functions in (2.3). We will work in the lowest-level approximation, in which we keep only the ground state tachyons in each twisted sector, and include bulk tachyon modes up to a level equal to the level of the highest twisted

[2] We set $\alpha' = 1$.

tachyon. Since the level of the bulk tachyon is given by[3] $\ell_u(p) = p^2/2$, this means that the momentum integrals will be truncated.

The gravitational coupling constant in flat space κ^2 is related to the orbifold gravtitational constant κ_N^2 as

$$\kappa_N^2 = \frac{\kappa^2}{N} \,. \tag{2.10}$$

One way to see this is to note that the integration region in the orbifold effective theory (1.1) is the the reduced space $\mathbb{C}/\mathbb{Z}_N$, whereas in the string field theory action (2.9) it is the covering space $\mathbb{C}$. The relative factor of N in (2.10) comes from the relative volumes of the two spaces.[4]

3. The Tachyon potential in $\mathbb{C}/\mathbb{Z}_4$

The conformal dimensions of the three twist fields are given by

$$h_{\sigma_1} = h_{\sigma_3} = \frac{3}{32} \qquad h_{\sigma_2} = \frac{1}{8} \,. \tag{3.1}$$

This gives a complex tachyon t of mass squared $m_t^2 = -29/8$ and level $\ell_t = 3/16$, and a real tachyon t' of mass squared $m_{t'}^2 = -7/2$ and level $\ell_{t'} = 1/4$. The lowest-level localized tachyon potential to cubic order is then given by

$$V_4(u,t,t') = V_{u^2} + V_{u^3} + V_{t^*t} + V_{t'^2} + V_{ut^*t} + V_{ut'^2} + V_{(t^2+t^{*2})t'} \,, \tag{3.2}$$

where V_{u^2} and V_{u^3} are given by (2.9), and the rest are given as follows

$$V_{t^*t} = -\frac{29}{8} t^* t \tag{3.3}$$

$$V_{t'^2} = -\frac{1}{2}\frac{7}{2} t'^2 \tag{3.4}$$

$$V_{ut^*t} = 4\kappa \mathcal{R}^{45/8} t^* t \int \frac{d^2p}{(2\pi)^2} \left[\mathcal{R}^2 \delta\left(\frac{1}{4}\right)\right]^{-\frac{1}{4}p^2} u(p) \tag{3.5}$$

$$V_{ut'^2} = 2\kappa \mathcal{R}^{11/2} t'^2 \int \frac{d^2p}{(2\pi)^2} \left[\mathcal{R}^2 \delta\left(\frac{1}{2}\right)\right]^{-\frac{1}{4}p^2} u(p) \tag{3.6}$$

$$V_{(t^2+t^{*2})t'} = \frac{\sqrt{2}\kappa}{\pi} \frac{\Gamma^2\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{2}\right)} \mathcal{R}^{43/8} (t^2 + t^{*2}) t' \,, \tag{3.7}$$

and $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(3/4) = 1.22542$. Following [11] we eliminate κ by rescaling quantities as follows

$$p_a = 2\xi_a \,, \quad x_a = \frac{1}{2} r_a \,, \qquad a = 1,2$$

[3] The level is defined as $\ell \equiv L_0 + \overline{L}_0 + 2$.

[4] We would like to thank the referee of this paper for turning our attention to this point, and to Yuji Okawa and Barton Zwiebach for this simple explanation. The relation (2.10) is also implied in [20].

$$u(\xi) = 4\kappa u(p)\,, \quad t \to \frac{1}{\kappa}t\,, \quad t' \to \frac{1}{\kappa}t'\,, \tag{3.8}$$

and defining the normalized dimensionless potential as[5]

$$f_4(u,t,t') \equiv \frac{\kappa^2 V_4(u,t,t')}{2\pi(1-\frac{1}{4})}\,. \tag{3.9}$$

This gives

$$\begin{aligned} f_4(u,t,t') =& -\frac{7}{6\pi}t'^2 - \frac{29}{12\pi}t^*t - \frac{1}{3\pi}\int \frac{d^2\xi}{(2\pi)^2}u(-\xi)(1-\xi^2)u(\xi) + \\ &+\frac{8}{3\pi}\mathcal{R}^{45/8}t^*t\int \frac{d^2\xi}{(2\pi)^2}\left[\mathcal{R}^2\delta\left(\frac{1}{2}\right)\right]^{-\xi^2}u(\xi) + \\ &+\frac{4}{3\pi}\mathcal{R}^{11/2}t'^2\int \frac{d^2\xi}{(2\pi)^2}\left[\mathcal{R}^2\delta\left(\frac{1}{4}\right)\right]^{-\xi^2}u(\xi) + \beta(t^2+t^{*2})t' + \\ &+\frac{1}{9\pi}\int\prod_{i=1}^{3}\left[\frac{d^2\xi_i}{(2\pi)^2}\mathcal{R}^{2(1-\xi_i^2)}u(\xi_i)\right](2\pi)^2\delta^{(2)}(\xi_1+\xi_2+\xi_3)\,, \end{aligned} \tag{3.10}$$

where

$$\beta \equiv \frac{2\sqrt{2}}{3\pi^2}\frac{\Gamma^2\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{2}\right)}\mathcal{R}^{43/8} = 0.330244\,. \tag{3.11}$$

The predicted values at the critical points are

$$f_4 = 4\widetilde{f}_4 = -4, -4/3, -4/9\,, \tag{3.12}$$

corresponding to flat space, $\mathbb{C}/\mathbb{Z}_2$ and $\mathbb{C}/\mathbb{Z}_3$, respectively.

In our lowest-level approximation we have only included the ground state tachyons from the twisted sectors. It is therefore reasonable to include only bulk tachyon modes up to the level of the higher of the two twisted tachyons. This imposes a cutoff on the momentum integrals of the bulk tachyon given by $\xi_* = \sqrt{\ell_{t'}/2} = 1/(2\sqrt{2})$.

4. The critical points

Varying the potential (3.10) with respect to the bulk and twisted tachyons we obtain

$$\begin{aligned} u(\xi) =& \frac{1}{1-\xi^2}\left\{2\mathcal{R}^{\frac{11}{2}}t'^2\left[\mathcal{R}^2\delta\left(\frac{1}{2}\right)\right]^{-\xi^2} + 4\mathcal{R}^{\frac{45}{8}}t^*t\left[\mathcal{R}^2\delta\left(\frac{1}{4}\right)\right]^{-\xi^2}\right\} + \\ &+\frac{1}{8\pi^2}\frac{\mathcal{R}^{2(1-\xi^2)}}{1-\xi^2}\int d^2\xi'\mathcal{R}^{2(2-\xi'^2-(\xi+\xi')^2)}u(\xi')u(-\xi'-\xi) \end{aligned} \tag{4.1}$$

$$t' = \frac{8}{7}t'\mathcal{R}^{\frac{11}{2}}\int\frac{d\xi^2}{(2\pi)^2}\left[\mathcal{R}^2\delta\left(\frac{1}{2}\right)\right]^{-\xi^2}u(\xi) + \frac{3\pi}{7}\beta(t^2+(t^*)^2) \tag{4.2}$$

[5]Note that f_N and $\widetilde{f}_N$ defined in (1.3) are related as $f_N = N\widetilde{f}_N$ because of (2.10).

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$$t = \frac{32}{29} t \mathcal{R}^{\frac{45}{8}} \int \frac{d\xi^2}{(2\pi)^2} \left[\mathcal{R}^2 \delta\left(\frac{1}{4}\right)\right]^{-\xi^2} u(\xi) + \frac{24\pi}{29} \beta t^* t' \,. \tag{4.3}$$

Before turning to the solutions let us derive some general properties of some of the critical points from these conditions. The trivial solution $u = t = t' = 0$ is the maximum and corresponds to the original $\mathbb{C}/\mathbb{Z}_4$. If $t' = 0$ (4.2) gives $t^2 + (t^*)^2 = 0$ and therefore $t = |t|(\pm 1 \pm i)/\sqrt{2}$. These four points are related by the $\mathbb{Z}_4$ symmetry of the potential. If both t and t' are non-vanishing (4.3) implies that t must be either real or imaginary.

Solving these conditions analytically is impossible due to the non-linear integral term in the equation for u, so we must resort to some sort of approximation method. One approach is to discretize the spectrum of the bulk tachyon by putting it in a box, and then solve the equations numerically, keeping a finite number of modes. This was the method employed in [11]. Our approach will be to solve the equations perturbatively, treating the quadratic term in u as a small perturbation. In comparing this approach to the one of [11] for $\mathbb{C}/\mathbb{Z}_2$ and $\mathbb{C}/\mathbb{Z}_3$, we find similar results.

To simplify notation we define

$$v(\xi) \equiv \mathcal{R}^{-2\xi^2} u(\xi) \qquad \lambda \equiv \frac{\mathcal{R}^6}{8\pi^2} = 0.060861 \tag{4.4}$$

$$\gamma(\xi) \equiv 2\mathcal{R}^{\frac{11}{2}} t'^2 \left[\delta\left(\frac{1}{2}\right)\right]^{-\xi^2} + 4\mathcal{R}^{\frac{45}{8}} t^* t \left[\delta\left(\frac{1}{4}\right)\right]^{-\xi^2} . \tag{4.5}$$

The equation for the bulk tachyon (4.1) then becomes

$$v(\xi) = \frac{\mathcal{R}^{-4\xi^2}}{1-\xi^2} \left[\gamma(\xi) + \lambda \int d^2\xi' v(\xi') v(\xi' + \xi)\right] . \tag{4.6}$$

Note that only $\gamma(\xi)$ depends on the specific orbifold. We would like to solve for $v(\xi)$ in terms of the twisted tachyons t and t' as a perturbative series in λ,

$$v(\xi, \lambda) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} v^{(n)}(\xi) \qquad v^{(n)}(\xi) = \frac{\partial^n}{\partial \lambda^n} v(\xi, \lambda)\Big|_{\lambda=0} . \tag{4.7}$$

At zeroth order

$$v^{(0)}(\xi) = \frac{\mathcal{R}^{-4\xi^2}}{1-\xi^2} \gamma(\xi) \,, \tag{4.8}$$

and the higher coefficients satisfy the recursion relation

$$v^{(n)}(\xi) = n \frac{\mathcal{R}^{-4\xi^2}}{1-\xi^2} \sum_{k=0}^{n-1} \binom{n-1}{k} \int d^2\xi' v^{(k)}(\xi') v^{(n-k-1)}(\xi + \xi') \,. \tag{4.9}$$

This can be also seen as an expansion in the number of integrals. The next two terms in the expansion are given by

$$v^{(1)}(\xi) = \frac{\mathcal{R}^{-4\xi^2}}{1-\xi^2} \int d^2\xi' \frac{\gamma(\xi')\gamma(\xi'+\xi) \mathcal{R}^{-4(\xi'^2 + (\xi+\xi')^2)}}{(1-\xi'^2)(1-(\xi'+\xi)^2)} \tag{4.10}$$

$$v^{(2)}(\xi) = \frac{2\mathcal{R}^{-4\xi^2}}{1-\xi^2}\int \frac{d^2\omega\, d^2\rho\, \gamma(\omega)\gamma(\rho)\gamma(\omega+\rho+\xi)\mathcal{R}^{-4(\omega^2+\rho^2+(\rho+\xi)^2+(\omega+\rho+\xi)^2)}}{(1-\omega^2)(1-\rho^2)(1-(\xi+\rho)^2)(1-(\omega+\rho+\xi)^2)}\,. \quad (4.11)$$

Therefore we can expand the effective potential for the twisted tachyons as a power series in λ as well,

$$f_4(t,t') = \sum_{n=0}^{\infty} \lambda^n f_4^{(n)}(t,t')\,. \quad (4.12)$$

At zeroth order the cubic bulk tachyon term doesn't contribute and we get

$$\begin{aligned} f_4^{(0)}(t,t') &= -\frac{7}{6\pi}t'^2 - \frac{29}{12\pi}t^*t + \beta(t^2+(t^*)^2)t' + \frac{1}{3\pi}\int \frac{d^2\xi}{(2\pi)^2}\gamma(\xi)v_0(\xi) \\ &= -\frac{7}{6\pi}t'^2 - \frac{29}{12\pi}t^*t + \beta(t^2+(t^*)^2)t' + (at^*t+bt'^2)^2 - ct^*tt'^2\,, \end{aligned} \quad (4.13)$$

where $\beta = 0.330244$, $a = 0.44631$, $b = 0.23264$ and $c = 0.00025$. There are four classes of critical points:

1. $(t,t')^{(0)}_{\mathbb{Z}_1} = (\pm 1.801, -1.722), (\pm 1.801i, 1.722)$ $\qquad f_4^{(0)} = -2.719$
2. $(t,t')^{(0)}_{\mathbb{Z}_2} = (\pm 0.983 \pm 0.983i, 0)$ $\qquad f_4^{(0)} = -0.743$
3. $(t,t')^{(0)}_{\mathbb{Z}'_2} = (0, \pm 1.852)$ $\qquad f_4^{(0)} = -0.637$
4. $(t,t')^{(0)}_{\mathbb{Z}_3} = (\pm 0.688, 0.718), (\pm 0.688i, -0.718)$ $\qquad f_4^{(0)} = -0.221$

The relative values of the tachyon potential at the critical points suggest that the first class of points corresponds to flat space, the second and third to $\mathbb{C}/\mathbb{Z}_2$ and the fourth to $\mathbb{C}/\mathbb{Z}_3$ (hence the labels). This can be verified by examining the spectrum of fluctuations around each kind of critical point, and counting the number of negative modes, i.e. tachyons (see appendix B). Comparing with the predicted values (3.12) we find that the minimum is at 68% of its predicted value of -4, the $\mathbb{C}/\mathbb{Z}_2$ points are at 55% and 48% (for the two classes) of their predicted value of $-4/3$, and the $\mathbb{C}/\mathbb{Z}_3$ points are at 50% of their predicted value of $-4/9$. The results for the first four orders in perturbation theory are summarized in table 1 (for more details see appendix A). The depth of the minimum dramatically decreases: we get 25% of the predicted value for flat space at third order. The depths of the $\mathbb{C}/\mathbb{Z}_2$ and $\mathbb{C}/\mathbb{Z}_3$ points also decrease with the order of the calculation, but not nearly as much. We end up with 39% and 34% agreement for the two classes of $\mathbb{C}/\mathbb{Z}_2$ points, and 46% for the $\mathbb{C}/\mathbb{Z}_3$ points.

The degeneracy of the potential within each class of critical points is a consequence of the $\mathbb{Z}_4$ symmetry of the orbifold theory. The critical points are located roughly at the vertices, edges and faces of a tetrahedron in the three-dimensional twisted-tachyon space (figure 1). The four vertices correspond to flat space, the six edges to $\mathbb{C}/\mathbb{Z}_2$, and the four faces to $\mathbb{C}/\mathbb{Z}_3$. The center of the tetrahedron corresponds to the original orbifold $\mathbb{C}/\mathbb{Z}_4$. In figure 2 we show contour plots of the tachyon potential on two different two-

Order	Vacuum	$\mathbb{C}/\mathbb{Z}_2$ quartet	$\mathbb{C}/\mathbb{Z}_2$ doublet	$\mathbb{C}/\mathbb{Z}_3$
0	-2.71904	-0.742683	-0.637051	-0.221467
1	-1.45297	-0.587339	-0.505818	-0.208521
2	-1.18091	-0.543738	-0.46922	-0.205589
3	-1.06427	-0.52420	-0.452928	-0.204618
Predicted	-4	-4/3	-4/3	-4/9

Table 1: The calculated values of f_4 in first orders of perturbation series.

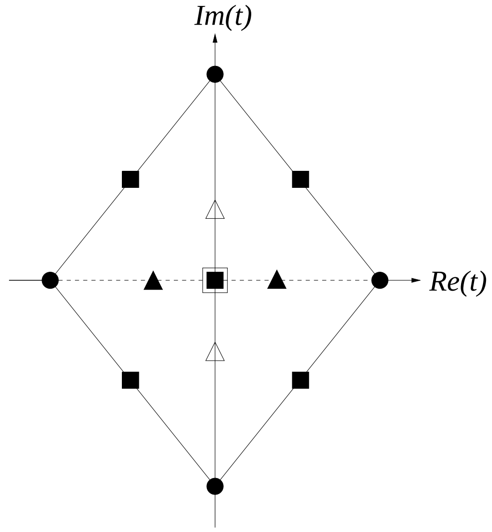

Figure 1: A schematic picture of the critical points of $f_4(t, t')$ projected onto the t-plane. The circles are the minima, i.e. flat space, the squares are the $\mathbb{C}/\mathbb{Z}_2$ points, and the triangles are the $\mathbb{C}/\mathbb{Z}_3$ points.

dimensional sections, revealing the tetrahedral structure. The tetrahedron defined by the four vertices is not a regular one; it has four edges (two pairs of opposing edges) of one length, and two of another length. Its symmetry group is D_4, which is a subgroup of the full symmetry of the regular tetrahedron T_d. The quantum symmetry of the orbifold $\mathbb{Z}_4$ is the maximal abelian subgroup of D_4. The four equal edges do not mix with the other two under $\mathbb{Z}_4$ or D_4, which explains the two classes of $\mathbb{C}/\mathbb{Z}_2$ points. Ultimately we know that since there is a unique $\mathbb{C}/\mathbb{Z}_2$ orbifold all the $\mathbb{C}/\mathbb{Z}_2$ points must be the same. An interesting question is whether the symmetry of the tachyon potential gets enhanced to the full tetrahedral group T_d at higher level, thereby guaranteeing a six-fold degeneracy of $\mathbb{C}/\mathbb{Z}_2$ points.

To check that our approximation scheme is as good as that of [11] we applied it to $\mathbb{C}/\mathbb{Z}_2$ and $\mathbb{C}/\mathbb{Z}_3$ as well. Using the perturbative technique to third order we find the minimum for $\mathbb{C}/\mathbb{Z}_2$ with $f_2 = -0.6794$, compared to $f_2 = -0.7202$ in [11]. For $\mathbb{C}/\mathbb{Z}_3$ the perturbative expansion to third order gives $f_3 = -0.9882, -0.3240$ for the minimum and $\mathbb{C}/\mathbb{Z}_2$ points respectively, compared to $f_3 = -0.9889, -0.3356$ in [11].

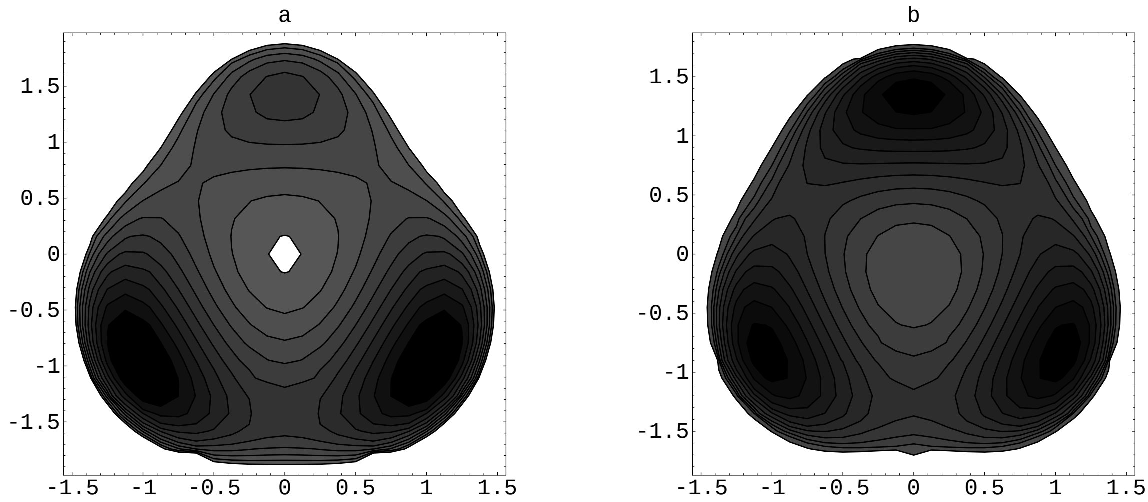

Figure 2: Contour plots of $f_4(t,t')$ on (a) the slice $\mathrm{Im}(t)=0$, (b) one of the faces of the tetrahedron. The first plot shows the maximum $f_4=0$ at the origin, two of the minima at $(\pm 1.045, -0.970)$, two $\mathbb{C}/\mathbb{Z}_2$ saddle points at $(0, \pm 1.441)$ (one is a local minimum on this cross section), and two $\mathbb{C}/\mathbb{Z}_3$ points at $(\pm 0.639, 0.671)$. The second plot shows (roughly) a $\mathbb{C}/\mathbb{Z}_3$ point (a local maximum on this cross section) at the origin, three minima, and three $\mathbb{C}/\mathbb{Z}_2$ saddle points.

4.1 A consistency check

Not all of the critical points are independent. The $\mathbb{C}/\mathbb{Z}_2$ doublet of the $\mathbb{C}/\mathbb{Z}_4$ potential is in fact related to the minimum of $\mathbb{C}/\mathbb{Z}_2$ potential (at lowest level). Consider the $\mathbb{C}/\mathbb{Z}_4$ tachyon potential with the complex tachyon vanishing, $V_4(0,t')$. The real tachyon t' corresponds to the twist 2 field σ_2 (the middle twisted sector), which has the same conformal dimension as the twist 1 field σ_1 of the $\mathbb{C}/\mathbb{Z}_2$ orbifold, since in both cases $k/N = 1/2$. This means all correlation functions involving this field and the untwisted fields are equal in the two theories. In particular the tachyon potentials are equal[6]

$$V_4(0,t') = V_2(t')\,. \tag{4.14}$$

The minimum of the potential on the right hand side corresponds to flat space in the $\mathbb{C}/\mathbb{Z}_2$ theory. On the other hand minimizing the potential on the left hand side with respect to t' gives the $\mathbb{C}/\mathbb{Z}_2$ doublet in the $\mathbb{C}/\mathbb{Z}_4$ theory. The equality above could therefore provide an alternative prediction for these $\mathbb{C}/\mathbb{Z}_2$ points. In terms of the normalized potential $\widetilde{f}_4$ the original prediction (1.4) for (all) the $\mathbb{C}/\mathbb{Z}_2$ points is $\widetilde{f}_4 = -1/3$, and the one that follows from (4.14) is $\widetilde{f}'_4 = (1/3)\widetilde{f}_2 = -1/3$, so the predictions agree.

We can generalize this observation for higher orbifolds. Any $\mathbb{C}/\mathbb{Z}_N$ orbifold for which N is a product exhibits potential multiple predictions for some of the critical points. Consider for example $\mathbb{C}/\mathbb{Z}_{pq}$. The twist qk field σ_{qk} has the same dimensions as the twist k field in $\mathbb{C}/\mathbb{Z}_p$. Therefore setting all tachyons but the ones associated with the twist fields σ_{qk}

[6] For the purpose of comparing the potentials in the two theories we use t' to denote both the twist 2 field in $\mathbb{C}/\mathbb{Z}_4$ and the twist 1 field in $\mathbb{C}/\mathbb{Z}_2$.

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for $k = 1, \ldots, p-1$ to zero, the tachyon potential V_{pq} becomes the same as the tachyon potential V_p of the $\mathbb{C}/\mathbb{Z}_p$ theory,

$$V_{pq}(0, \ldots, 0, t_q = t_1', 0, \ldots, 0, t_{2q} = t_2', \ldots) = V_p(t_1', t_2', \ldots) . \tag{4.15}$$

We now want to show that the critical points with respect to t_{qk} of the reduced potential on the left hand side are also critical points of the full potential V_{pq} with respect to all the t_k. The only possible problems can come from terms in the potential linear in one of the vanishing tachyons, $t_s \prod_{i=1}^{n-1} t_{qk_i}$, where s is not an integer multiple of q. Varying with respect to t_s one could arrive at a non vanishing result for the vanishing tachyon t_s. However such terms are forbidden by the quantum symmetry of the orbifold Z_{pq}, since this requires $s + q\sum_i k_i = npq$ for some integer n, and therefore $s = q(np - \sum_i k_i)$. Thus the critical points of V_{pq} with $t_s = 0$ ($s \neq qk$) are identical to the critical points of V_p. This provides a possible alternative prediction for each of these critical points of V_{pq}. The points in question correspond to the orbifolds $\mathbb{C}/\mathbb{Z}_{kq}$ with $k = 1, \ldots, p-1$. The original prediction for the normalized potential (1.4) is

$$\widetilde{f}_{pq}(\mathbb{C}/\mathbb{Z}_{kq}) = -\frac{\frac{1}{k} - \frac{1}{p}}{q - \frac{1}{p}} , \tag{4.16}$$

and the one that follows from the relation to V_p is

$$\widetilde{f}'_{pq}(\mathbb{C}/\mathbb{Z}_{kq}) = \frac{\kappa_{pq}^2}{\kappa_p^2} \frac{1 - \frac{1}{p}}{1 - \frac{1}{pq}} \widetilde{f}_p(\mathbb{C}/\mathbb{Z}_k) = -\frac{\frac{1}{k} - \frac{1}{p}}{q - \frac{1}{p}} , \tag{4.17}$$

in agreement with $\widetilde{f}_{pq}(\mathbb{C}/\mathbb{Z}_{kq})$.

5. Conclusions

Extending the work of [11], we have computed the potential for localized tachyons in the bosonic string orbifold $\mathbb{C}/\mathbb{Z}_4$ using a lowest-level truncation of closed string field theory to cubic order. Our computation provides additional evidence for the conjecture of [1], that the critical points of the tachyon potential correspond to lower-order orbifolds, and in particular that the minimum is flat space. The results are consistent with those of [11] for the $\mathbb{C}/\mathbb{Z}_2$ and $\mathbb{C}/\mathbb{Z}_3$ theories. We also believe that these results can be taken as support for the general method of level-truncation, together with string-vertex truncation, in closed string field theory.

There are a couple of interesting puzzles that require further study. The first is that there appear to be two classes of $\mathbb{C}/\mathbb{Z}_2$ critical points, with different values of the tachyon potential. The $\mathbb{Z}_4$ (or D_4) symmetry of the tachyon potential guarantees the degeneracy within each class, but not between the two classes. The latter requires the full tetrahedral symmetry T_d. The question is whether this symmetry is restored at higher level, or whether at higher level the two classes of points become accidentally degenerate. The former scenario is more appealing, since it would also explain the quantum symmetries of the different critical points. The tetrahedral group T_d contains $\mathbb{Z}_4, \mathbb{Z}_3$ and $\mathbb{Z}_2$, corresponding

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to the abelian subgroups preserving the center, a face, and a pair of opposite edges, respectively. We could therefore identify these groups as the unbroken symmetries of $\mathbb{C}/\mathbb{Z}_4$, $\mathbb{C}/\mathbb{Z}_3$ and $\mathbb{C}/\mathbb{Z}_2$, respectively.

Another interesting puzzle raised by the calculation of localized closed string tachyon potentials is the possibility of stable non-uniform tachyon configurations, i.e. tachyonic solitons. For example in the $\mathbb{C}/\mathbb{Z}_2$ theory the effective potential for the twisted tachyon has two degenerate minima [11], and should therefore admit a kink solution. In the analogous situation for open strings, namely an unstable (non-BPS) Dp-brane in type-II (or type 0) string theory, the tachyonic kink corresponds to a stable (BPS) D$(p-1)$-brane [21, 13]. The topological stability of the kink configuration in the orbifold implies that it corresponds to a closed string background which is stable (modulo the usual bulk tachyon). This appears to be a new background of the bosonic string, which is as stable as flat space. It would be interesting to study this background further.

Acknowledgments

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A. Details of the perturbative calculation

Here we give the details of the calculation of the first few orders in the perturbative expansion of the twisted tachyon effective potential. Recall that the first couple of terms in the expansion for the scaled bulk tachyon $v(\xi)$ are given by

$$\begin{aligned} v^{(1)}(\xi) &= \frac{v^{(0)}(\xi)}{\gamma(\xi)} \int d^2\xi_1 v^{(0)}(\xi_1) v^{(0)}(\xi_1+\xi)\,, \\ v^{(2)}(\xi) &= 4\frac{v^{(0)}(\xi)}{\gamma(\xi)} \int d^2\xi_1 v^{(1)}(\xi_1) v^{(0)}(\xi_1+\xi)\,. \end{aligned} \tag{A.1}$$

The tachyon potential is expanded as

$$f_4 = \sum_{k=0}^{\infty} \lambda^k f_4^{(k)}\,. \tag{A.2}$$

The zeroth order term was given in the bulk of the paper (4.13). The first order term is given by

$$\begin{aligned} f_4^{(1)}(t,t') &= \frac{8\pi}{9} \int \frac{d^2\xi_1 d^2\xi_2}{(2\pi)^4} v^{(0)}(\xi_1) v^{(0)}(\xi_2) v^{(0)}(\xi_1+\xi_2) \\ &= a_{30}^{(1)}(tt^*)^3 + a_{03}^{(1)} t'^6 + a_{21}^{(1)}(tt^*)^2 t'^2 + a_{12}^{(1)} t'^4 (tt^*)\,, \end{aligned} \tag{A.3}$$

where

$$a_{12}^{(1)} = \frac{6(2\mathcal{R}^{11/2})^2(4\mathcal{R}^{45/8})}{9\pi}\eta(1/2,1/4) = 0.4846\,, \quad a_{30}^{(1)} = \frac{2(4\mathcal{R}^{45/8})^3}{9\pi}\eta(1/4,1/4) = 0.5940$$

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$$a_{21}^{(1)} = \frac{6(4\mathcal{R}^{45/8})^2(2\mathcal{R}^{11/2})}{9\pi}\eta(1/4,1/2) = 0.9750\,, \quad a_{03}^{(1)} = \frac{2(2\mathcal{R}^{11/2})^3}{9\pi}\eta(1/2,1/2) = 0.0887$$

and where we have defined

$$\eta(a,b) \equiv \int \frac{d^2\xi d^2\xi'}{(2\pi)^2}\frac{(\mathcal{R}^4\delta(a))^{-\xi^2-\xi'^2}(\mathcal{R}^4\delta(b))^{-(\xi'+\xi)^2}}{(1-\xi^2)(1-\xi'^2)(1-(\xi'+\xi)^2)}\,. \tag{A.4}$$

The second order term is given by

$$\begin{aligned} f_4^{(2)}(t,t') &= -\frac{1}{3\pi}\int \frac{d^2\xi}{(2\pi)^2}\frac{\gamma(\xi)}{v^{(0)}(\xi)}\left(v^{(1)}(\xi)\right)^2 + \frac{8\pi}{3}\int \frac{d^2\xi_1 d^2\xi_2}{(2\pi)^4}v^{(1)}(\xi_1)v^{(0)}(\xi_2)v^{(0)}(\xi_1+\xi_2) \\ &= \frac{1}{3\pi}\int \frac{d^2\xi}{(2\pi)^2}\frac{\gamma(\xi)}{v^{(0)}(\xi)}\left(v^{(1)}(\xi)\right)^2 \\ &= a_{40}^{(2)}(tt^*)^4 + a_{04}^{(2)}t'^8 + a_{31}^{(2)}(tt^*)^3t'^2 + a_{13}^{(2)}t'^6(tt^*) + a_{22}^{(2)}t'^4(tt^*)^2\,, \end{aligned} \tag{A.5}$$

where

$$\begin{aligned} a_{13}^{(2)} &= \frac{(2\mathcal{R}^{11/2})^3(4\mathcal{R}^{45/8})}{3\pi}\tilde\eta_1(1,3) = 2.4433\,, \quad a_{40}^{(2)} = \frac{(4\mathcal{R}^{45/8})^4}{3\pi}\tilde\eta_1(4,0) = 3.9897 \\ a_{31}^{(2)} &= \frac{(4\mathcal{R}^{45/8})^3(2\mathcal{R}^{11/2})}{3\pi}\tilde\eta_1(3,1) = 8.5329\,, \quad a_{04}^{(2)} = \frac{(2\mathcal{R}^{11/2})^4}{3\pi}\tilde\eta_1(0,4) = 0.3271 \\ a_{22}^{(2)} &= \frac{(2\mathcal{R}^{11/2})^2(4\mathcal{R}^{45/8})^2}{3\pi}\tilde\eta_1(2,2) = 6.8472\,, \end{aligned} \tag{A.6}$$

and where we have defined

$$\tilde\eta_1(k,l) \equiv \frac{1}{k!l!}\sum_{P\in S_4}\eta\Big(P(\underbrace{1/4,\ldots,1/4}_{k},\underbrace{1/2,\ldots,1/2}_{l})\Big)\,, \quad k+l=4\,, \tag{A.7}$$

$$\eta(a,b,c,d) \equiv \int \frac{d^2\xi\, d^2\omega\, d^2\rho}{(2\pi)^2}\frac{(\mathcal{R}^4\delta(a))^{-\omega^2}(\mathcal{R}^4\delta(c))^{-(\omega+\xi)^2}(\mathcal{R}^4\delta(b))^{-\rho^2}(\mathcal{R}^4\delta(d))^{-(\rho+\xi)^2}\mathcal{R}^{-4\xi^2}}{(1-\xi^2)(1-\omega^2)(1-(\omega+\xi)^2)(1-\rho^2)(1-(\rho+\xi)^2)}\,.$$

The third order term is given by

$$\begin{aligned} f_4^{(3)}(t,t') &= -\frac{1}{3\pi}\int \frac{d^2\xi}{(2\pi)^2}\frac{\gamma(\xi)}{v^{(0)}(\xi)}v^{(1)}(\xi)v^{(2)}(\xi) + \frac{8\pi}{3}\int \frac{d^2\xi_1 d^2\xi_2}{(2\pi)^4}v^{(1)}(\xi_1)v^{(1)}(\xi_2)v^{(0)}(\xi_1+\xi_2) \\ &\quad + \frac{4\pi}{3}\int \frac{d^2\xi_1 d^2\xi_2}{(2\pi)^4}v^{(2)}(\xi_1)v^{(0)}(\xi_2)v^{(0)}(\xi_1+\xi_2) \\ &= \frac{1}{6\pi}\int \frac{d^2\xi}{(2\pi)^2}v^{(1)}(\xi)v^{(2)}(\xi)\frac{\gamma(\xi)}{v^{(0)}(\xi)} \\ &= a_{50}^{(3)}(tt^*)^5 + a_{05}^{(3)}t'^{10} + a_{41}^{(3)}(tt^*)^4t'^2 + a_{14}^{(3)}t'^8(tt^*) + a_{32}^{(3)}t'^4(tt^*)^3 + a_{23}^{(3)}t'^6(tt^*)^2\,, \end{aligned} \tag{A.8}$$

where

$$\begin{aligned} a_{14}^{(3)} &= \frac{2(2\mathcal{R}^{11/2})^4(4\mathcal{R}^{45/8})}{3\pi}\tilde\eta_2(1,4) = 14.9425\,, \quad a_{50}^{(3)} = \frac{2(4\mathcal{R}^{45/8})^5}{3\pi}\tilde\eta_2(5,0) = 35.7330 \\ a_{41}^{(3)} &= \frac{2(4\mathcal{R}^{45/8})^4(2\mathcal{R}^{11/2})}{3\pi}\tilde\eta_2(4,1) = 96.0050\,, \quad a_{05}^{(3)} = \frac{2(2\mathcal{R}^{11/2})^5}{3\pi}\tilde\eta_2(0,5) = 1.6079 \end{aligned}$$

flat space			
order	t	t'	f_4
0	1.8006	-1.7216	-2.7190
1	1.2708	-1.1994	-1.4530
2	1.1192	-1.0452	-1.1809
3	1.0451	-0.9696	-1.0643

$\mathbb{C}/\mathbb{Z}_3$			
order	t	t'	f_4
0	0.6876	0.7177	-0.2215
1	0.6533	0.6844	-0.2085
2	0.6434	0.6751	-0.2056
3	0.6394	0.6713	-0.2046

$\mathbb{C}/\mathbb{Z}_2$ doublet			
order	t	t'	f_4
0	0	1.8523	-0.6371
1	0	1.5804	-0.5058
2	0	1.4876	-0.4692
3	0	1.4407	-0.4529

$\mathbb{C}/\mathbb{Z}_2$ quartet			
order	t	t'	f_4
0	0.9826(1+i)	0	-0.7427
1	0.8362(1+i)	0	-0.5873
2	0.7860(1+i)	0	-0.5437
3	0.7604(1+i)	0	-0.5242

Table 2: Extremal points of the tachyon potential f_4 at each order of perturbation theory. There are additional points related to these by the $\mathbb{Z}_4$ symmetry.

$$a^{(3)}_{23} = \frac{2(2\mathcal{R}^{11/2})^3(4\mathcal{R}^{45/8})^2}{3\pi}\tilde{\eta}_2(2,3) = 55.5283$$
$$a^{(3)}_{32} = \frac{2(2\mathcal{R}^{11/2})^2(4\mathcal{R}^{45/8})^3}{3\pi}\tilde{\eta}_2(3,2) = 103.2366$$

and where we have defined

$$\tilde{\eta}_2(k,l) \equiv \frac{1}{k!l!}\sum_{P\in S_5}\eta\Big(P(\underbrace{1/4...1/4}_{k},\underbrace{1/2...1/2}_{l})\Big)\,, \qquad k+l=5$$
$$\eta(a,b,c,d,e) = \int\frac{d^2\xi\, d^2\xi'\, d^2\omega\, d^2\rho}{(2\pi)^2}\times \tag{A.9}$$
$$\times\frac{(\mathcal{R}^4\delta(a))^{-\rho^2}(\mathcal{R}^4\delta(b))^{-(\xi+\omega)^2}(\mathcal{R}^4\delta(c))^{-\omega^2}(\mathcal{R}^4\delta(d))^{-(\xi+\xi')^2}(\mathcal{R}^4\delta(e))^{-(\xi'+\rho)^2}\mathcal{R}^{-4(\xi^2+\xi'^2)}}{(1-\xi^2)(1-\xi'^2)(1-(\xi+\xi')^2)(1-\omega^2)(1-\rho^2)(1-(\xi+\omega)^2)(1-(\xi'+\rho)^2)}\,.$$

The positions and potential values of the critical points at each order are shown in table 2.

B. Fluctuation analysis at the critical points

Here we will analyze the spectrum of fluctuations around the critical points of the tachyon potential. In particular we will show that the number of tachyons agrees with the identification of each critical point with a particular orbifold. For simplicity we look only at the zeroth order potential (4.13) (this should not affect the number of tachyons, only the precise values of their masses). The quadratic terms in the fluctuations of the tachyons δt, $\delta t'$ and δu are given by

$$f_4^{(0)}(t+\delta t, t'+\delta t', u(t,t')+\delta u) = \left[-\frac{7}{6\pi}+(2ab-c)|t|^2+6b^2t'^2\right]\delta t'^2+ \tag{B.1}$$
$$+\frac{1}{4}\left[-\frac{29}{12\pi}+(2ab-c)t'^2+\frac{1}{2}a^2(3t_+^2+t_-^2)+2\beta t'\right]\delta t_+^2+$$

$$
\begin{aligned}
&+\frac{1}{4}\Big[-\frac{29}{12\pi}+(2ab-c)t'^2+\frac{1}{2}a^2(3t_-^2+t_+^2)-2\beta t'\Big]\delta t_-^2+\\
&+\Big[(2ab-c)t_+t'+\beta t_+\Big]\delta t_+\delta t'+\\
&+\Big[(2ab-c)t_-t'-\beta t_-\Big]\delta t_-\delta t'+\\
&+\frac{1}{2}a^2t_+t_-\delta t_+\delta t_- -\frac{1}{3\pi}\int\frac{d^2\xi}{(2\pi)^2}\delta u(-\xi)(1-\xi^2)\delta u(\xi)\,,
\end{aligned}
$$

where $t_+ \equiv 2\,\mathrm{Re}(t)$ and $t_- \equiv 2\,\mathrm{Im}(t)$. We see that there is always at least one tachyonic mode coming from the bulk tachyon δu. Let us analyze each of the four classes of critical points in turn.

1. Minimum: $(t,t')=(1.801,-1.722)$. Diagonalizing the mass matrix of the fluctuations we find

$$m_1^2 = 0.4071\,,\ m_2^2 = 0.1572\,,\ m_3^2 = 1.2678\,, \tag{B.2}$$

corresponding respectively to δt_-, $(0.998\delta t_+ + 0.062\delta t')$ and $(-0.062\delta t_+ + 0.998\delta t')$. There are no tachyons (other than the bulk tachyon), which is consistent with the identification of this point with flat space.

2. $\mathbb{C}/\mathbb{Z}_2$ quartet: $(t,t')=(0.983(1+i),0)$. The eigenvalues of the mass matrix are given by

$$m_1^2 = -1.0060\,,\ m_2^2 = 0.8429\,,\ m_3^2 = 0.5769 \tag{B.3}$$

corresponding to the eigenvectors $(-0.529\delta t_+ + 0.529\delta t_- + 0.663\delta t')$, $(0.469\delta t_+ - 0.469\delta t_- + 0.748\delta t')$ and $(0.707\delta t_+ + 0.707\delta t_-)$. There is a single tachyonic mode, as one expects from the twisted sector of $\mathbb{C}/\mathbb{Z}_2$.

3. $\mathbb{C}/\mathbb{Z}_2$ doublet: $(t,t')=(0,1.852)$. Here there is no mixing between δt and $\delta t'$, and we simply get

$$m_{\delta t'}^2 = 0.7427\,,\ m_{\delta t_+}^2 = 0.2914\,,\ m_{\delta t_-}^2 = -0.3203\,, \tag{B.4}$$

Again there is just one tachyon, which is consistent with $\mathbb{C}/\mathbb{Z}_2$.

4. $\mathbb{C}/\mathbb{Z}_3$: $(t,t')=(0.688,0.718)$. Here we get

$$m_1^2 = -0.2606\,,\ m_2^2 = 0.4291\,,\ m_3^2 = -0.5116\,, \tag{B.5}$$

corresponding respectively to δt_-, $(0.754\delta t_+ + 0.657\delta t')$ and $(-0.657\delta t_+ + 0.754\delta t')$. We find two tachyonic modes, as there should be for $\mathbb{C}/\mathbb{Z}_3$.

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References

[1] A. Adams, J. Polchinski and E. Silverstein, *Don't panic! closed string tachyons in ale space-times*, *J. High Energy Phys.* **10** (2001) 029 [hep-th/0108075].

[2] Y. Michishita and P. Yi, *D-brane probe and closed string tachyons*, *Phys. Rev.* **D 65** (2002) 086006 [hep-th/0111199].

[3] C. Vafa, *Mirror symmetry and closed string tachyon condensation*, hep-th/0111051.

[4] J.A. Harvey, D. Kutasov, E.J. Martinec and G.W. Moore, *Localized tachyons and RG flows*, hep-th/0111154.

[5] M. Headrick, S. Minwalla and T. Takayanagi, *Closed string tachyon condensation: an overview*, *Class. Quant. Grav.* **21** (2004) S1539–S1565 [hep-th/0405064].

[6] R. Gregory and J.A. Harvey, *Spacetime decay of cones at strong coupling*, *Class. and Quant. Grav.* **20** (2003) L231 [hep-th/0306146].

[7] M. Headrick, *Decay of C/Z_N: exact supergravity solutions*, *J. High Energy Phys.* **03** (2004) 025 [hep-th/0312213].

[8] S. Sarkar and B. Sathiapalan, *Closed string tachyons on C/Z_N*, hep-th/0309029.

[9] A. Dabholkar, A. Iqubal and J. Raeymaekers, *Off-shell interactions for closed-string tachyons*, *J. High Energy Phys.* **05** (2004) 051 [hep-th/0403238].

[10] S.P. De Alwis and A.T. Flournoy, *Closed string tachyons and semi-classical instabilities*, *Phys. Rev.* **D 66** (2002) 026005 [hep-th/0201185].

[11] Y. Okawa and B. Zwiebach, *Twisted tachyon condensation in closed string field theory*, *J. High Energy Phys.* **03** (2004) 056 [hep-th/0403051].

[12] A. Sen, *Tachyon condensation on the brane antibrane system*, *J. High Energy Phys.* **08** (1998) 012 [hep-th/9805170].

[13] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[14] W. Taylor and B. Zwiebach, *D-branes, tachyons and string field theory*, hep-th/0311017.

[15] A. Dabholkar, *Tachyon condensation and black hole entropy*, *Phys. Rev. Lett.* **88** (2002) 091301 [hep-th/0111004].

[16] B. Zwiebach, *Closed string field theory: quantum action and the B-V master equation*, *Nucl. Phys.* **B 390** (1993) 33 [hep-th/9206084].

[17] A. Belopolsky and B. Zwiebach, *Off-shell closed string amplitudes: towards a computation of the tachyon potential*, *Nucl. Phys.* **B 442** (1995) 494 [hep-th/9409015];
A. Belopolsky, *Effective tachyonic potential in closed string field theory*, *Nucl. Phys.* **B 448** (1995) 245 [hep-th/9412106].

[18] N. Moeller, *Closed bosonic string field theory at quartic order*, *J. High Energy Phys.* **11** (2004) 018 [hep-th/0408067].

[19] L.J. Dixon, D. Friedan, E.J. Martinec and S.H. Shenker, *The conformal field theory of orbifolds*, *Nucl. Phys.* **B 282** (1987) 13.

[20] M. Headrick and J. Raeymaekers, *The large-N limit of C/Z_N and supergravity*, hep-th/0411148.

[21] A. Sen, *So(32) spinors of type-I and other solitons on brane- antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141].

JHEP01(2005)014